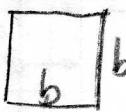


Sketch the base, set-up the integral to find the following volumes, then evaluate each integral by hand:

In 1 – 3, Find the volume of the solid formed with the given base and cross-sections perpendicular to the x-axis:

1) **Base:** $x^2 + y^2 = 25$



$$x^2 + y^2 = 25$$

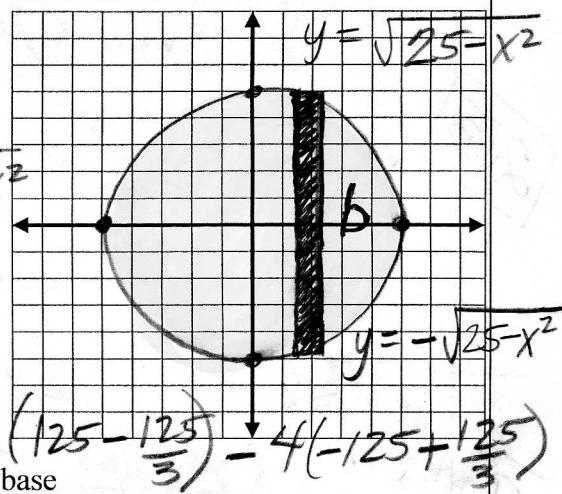
$$y^2 = 25 - x^2$$

$$y = \pm \sqrt{25 - x^2}$$

5 a) **Cross-Sections:** Squares

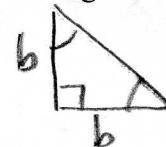
$$\int_{-5}^5 b^2 dx = \int_{-5}^5 (2\sqrt{25-x^2})^2 dx =$$

$$4 \int_{-5}^5 25 - x^2 dx = 4 \left[25x - \frac{1}{3}x^3 \right]_{-5}^5 = 4 \left(125 - \frac{125}{3} \right) - 4 \left(-125 + \frac{125}{3} \right)$$



b) **Cross-Sections:** Isosceles Right Triangles with one leg in the base

$$\int_{-5}^5 \frac{1}{2} (2\sqrt{25-x^2})^2 dx =$$



$$4(250 - \frac{250}{3}) =$$

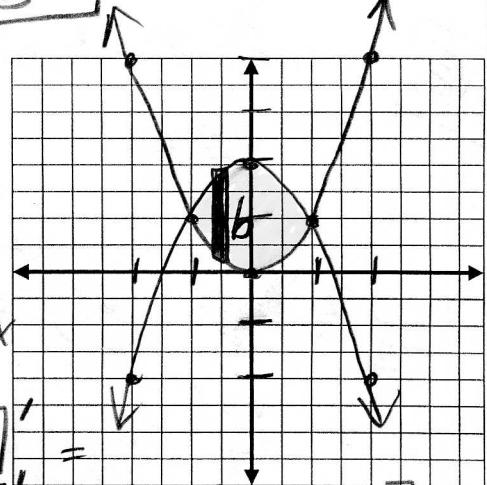
$$2 \int_{-5}^5 25 - x^2 dx = 2 \left(250 - \frac{250}{3} \right) = \boxed{\frac{1000}{3}}$$

$$\frac{3000}{3} - \frac{1000}{3} = \boxed{\frac{2000}{3}}$$

2) **Base:** Region bounded by $y = x^2$ and $y = 2 - x^2$

a) **Cross-Sections:** Rectangle whose height is twice the base

$$A = bh \quad \begin{cases} b = (2 - x^2) - x^2 = 2 - 2x^2 \\ h = 2b = 2(2 - 2x^2) \end{cases}$$

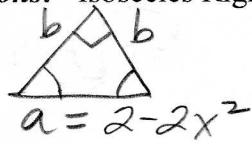


$$\int_{-1}^1 2(2 - 2x^2)^2 dx = 2 \int_{-1}^1 4 - 8x^2 + 4x^4 dx$$

$$8 \int_{-1}^1 1 - 2x^2 + x^4 dx = 8 \left[x - \frac{2}{3}x^3 + \frac{1}{5}x^5 \right]_1^{-1} =$$

b) **Cross-Sections:** Isosceles Right triangles with the hypotenuse in the base

$$A = \frac{1}{2} b^2$$



$$\frac{b}{1} = \frac{2 - 2x^2}{\sqrt{2}}$$

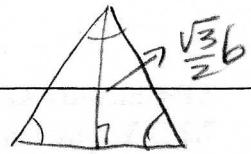
$$8 \left[1 - \frac{2}{3} + \frac{1}{5} \right] + 8 \left[1 + \frac{2}{3} + \frac{1}{5} \right] =$$

$$\int_{-1}^1 \frac{1}{2} \left(\frac{2 - 2x^2}{\sqrt{2}} \right)^2 dx = \frac{1}{4} \int_{-1}^1 4 - 8x^2 + 4x^4 dx =$$

$$\int_{-1}^1 1 - 2x^2 + x^4 dx = \boxed{\frac{16}{15}}$$

$$8 \left[2 - \frac{4}{3} + \frac{2}{5} \right] = 8 \left[2 - \frac{14}{15} \right] = 8 \left[\frac{16}{15} \right] = \boxed{\frac{128}{15}}$$

3) **Base:** Region bounded by $y = 2\sqrt{\sin x}$ in the interval $0 \leq x \leq \pi$ and the x -axis.

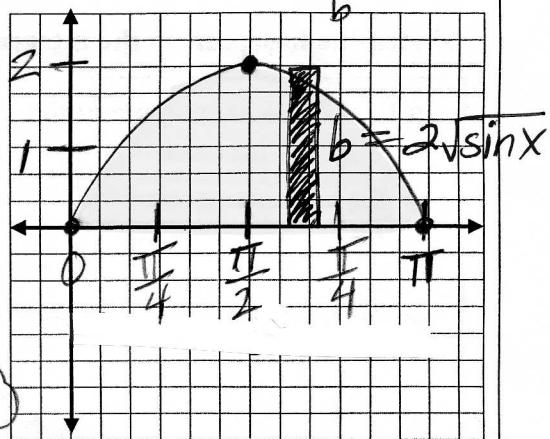


a) **Cross-Sections:** Equilateral Triangles $A = \frac{\sqrt{3}}{4}b^2$

$$\int_0^\pi \frac{\sqrt{3}}{4}(2\sqrt{\sin x})^2 dx = \sqrt{3} \int_0^\pi \sin x dx =$$

$$\sqrt{3} [-\cos x]_0^\pi = \sqrt{3} [-\cos \pi + \cos 0] =$$

b) **Cross-Sections:** Semicircles $\sqrt{3}[1+1] = 2\sqrt{3}$



$$A = \frac{1}{2}\pi r^2 = \frac{1}{2}\pi \left(\frac{b}{2}\right)^2$$

$$\int_0^\pi \frac{1}{2}\pi \left(\frac{2\sqrt{\sin x}}{2}\right)^2 dx = \frac{1}{2}\pi \int_0^\pi \sin x dx = \frac{1}{2}\pi [-\cos x]_0^\pi =$$

$$\frac{1}{2}\pi [-\cos \pi + \cos 0] = \frac{1}{2}\pi [1+1] = \pi$$

Find the volume of the solid formed with the given base and cross-sections perpendicular to the y -axis:

4) **Base:** $4x^2 + 9y^2 = 36$

$$\frac{x^2}{9} + \frac{y^2}{4} = 1$$

$$4x^2 = 36 - 9y^2$$

$$\sqrt{x^2} = \sqrt{\frac{36 - 9y^2}{4}}$$

a) **Cross-Sections:** Equilateral Triangles

$$A = \frac{\sqrt{3}}{4}b^2$$

$$x = \pm \sqrt{\frac{36 - 9y^2}{2}}$$

$$\int_{-2}^2 \frac{\sqrt{3}}{4} \left(\sqrt{\frac{36 - 9y^2}{2}}\right)^2 dy = \frac{\sqrt{3}}{4} \int_{-2}^2 (36 - 9y^2) dy =$$

$$x = -\sqrt{\frac{36 - 9y^2}{2}}, x = \sqrt{\frac{36 - 9y^2}{2}}$$

$$\frac{\sqrt{3}}{4} [36y - 3y^3]_{-2}^2 = \frac{\sqrt{3}}{4} [72 - 24] - \frac{\sqrt{3}}{4} [-72 + 24] =$$

b) **Cross-Sections:** 3, 4, 5 right triangles with the long leg in the base

$$A = \frac{1}{2}b \cdot h = \frac{1}{2}(b)\left(\frac{3}{4}b\right)$$

$$\frac{b}{4} = \frac{h}{3}$$

$$h = \frac{3}{4}b$$

$$\int_{-2}^2 \frac{3}{8} \left(\sqrt{\frac{36 - 9y^2}{2}}\right)^2 dy = \frac{3}{8} \int_{-2}^2 (36 - 9y^2) dy = \frac{3}{8} (96) =$$

36

$$\frac{\sqrt{3}}{4} [144 - 48] = \frac{\sqrt{3}}{4} (96) =$$

24\sqrt{3}

